Invertibilità

$f'(x) > 0$, $f \in C(D_f)$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$\sqrt[n]{z^n} = z \cdot \sqrt[n]{1}$$

Complessi

$$|z| = |\bar{z}| = \sqrt{a^2+b^2}$$

$$z = \rho(\cos\theta + i\sin\theta) \begin{cases} \cos\theta = a/|z| \\ \sin\theta = b/|z| \end{cases} = \rho e^{i\theta}$$

$$z^n = \rho^n(\cos(n\theta) + i\sin(n\theta))$$

$$\sqrt[n]{z} = \rho^{1/n}\left(\cos\left(\frac{\theta+2k\pi}{n}\right) + i\sin\left(\frac{\theta+2k\pi}{n}\right)\right) = \rho^{1/n} e^{i\frac{\theta+2k\pi}{n}}$$

$k = 0, 1, \dots, n-1$

Discontinuità

1° Specie: $\exists$ finiti $\lim_{x\to x_0^+} f(x) \neq \lim_{x\to x_0^-} f(x)$

2° Specie: se $\lim_{x\to x_0^+} f(x)$ o $\lim_{x\to x_0^-} f(x)$ $\nexists$ o non finito

Eliminabile: $\lim_{x\to x_0} f(x) = l = f(x_0)$ o $\neq f(x_0)$

$\sin, \cos(\alpha)$: $|f(x)| \le 1$

$$f(x)^{g(x)^{h(x)}} = e^{h(x)\ln(g(x))}$$

$$\operatorname{arctg} z + \operatorname{arctg}\frac{1}{z} = \begin{cases} \pi/2 & z > 0 \\ -\pi/2 & z < 0 \end{cases}$$

Serie

Radice

$\lim_{n\to+\infty} \sqrt[n]{|a_n|} = \alpha \in [0,+\infty) \cup \{+\infty\}$

- $\alpha \in [0,1) \Rightarrow \sum a_n$ conv. assolutamente
- $\alpha > 1 \Rightarrow \sum |a_n|$ diverge e $\sum a_n$ non conv.
- $\alpha = 1 \Rightarrow$???

Rapporto

$a_n \neq 0$ $\forall n \ge n_0$ e $\lim_{n\to+\infty} \left|\frac{a_{n+1}}{a_n}\right| = \alpha \in [0,+\infty) \cup \{+\infty\}$

- $\alpha \in [0,1) \Rightarrow \sum a_n$ conv. ass
- $\alpha > 1 \Rightarrow \sum |a_n|$ div e $\sum a_n$ non conv.
- $\alpha = 1 \Rightarrow$???

$$\sum \frac{1}{n^\alpha \log^\beta n} \text{ conv } \begin{cases} \alpha > 1 \\ \alpha = 1 \text{ e } \beta > 1 \end{cases}$$

Uniforme Continuità

- $f \in C(A)$, $A \subset \mathbb{R} \Rightarrow f$ U.C. in A, A chiuso e limitato (Heine-Cantor)
- f U.C. in $(a,b] \Leftrightarrow \lim_{x\to a^+} f(x) = l \in \mathbb{R}$
- $\left.\begin{array}{l} f \in C([a,+\infty)) \\ \lim_{x\to+\infty} f(x) = l \in \mathbb{R} \end{array}\right\} \Rightarrow f$ U.C. in $[a,+\infty)$
- $\left.\begin{array}{l} f \in C((a,+\infty)) \\ \lim_{x\to a^+} f(x) = l \in \mathbb{R} \\ \lim_{x\to+\infty} f(x) = l_1 \in \mathbb{R} \end{array}\right\} \Rightarrow f$ U.C. in $(a,+\infty)$
- $\left.\begin{array}{l} f \text{ derivabile in } I \subseteq \mathbb{R} \\ \exists M > 0 : |f'(x)| < M \ \forall x \in I \end{array}\right\} \Rightarrow f$ U.C. in I

Integrali

F esiste sse f è integrabile:

- $\lim_{x\to a} f(x) = l$ finito
- $\lim_{x\to a} f(x) = 1/x^\alpha$ con $\alpha < 1$
- $\lim_{x\to\infty} f(x) = 1/x^\alpha$ con $\alpha > 1$

$$\frac{1}{|x|^\alpha \cdot |\log|x||^\beta} \begin{cases} U(\infty): \text{conv. } \alpha > 1, \forall\beta; \ \alpha = 1, \beta > 1 \\ U(0): \text{conv. } \alpha < 1, \forall\beta; \ \alpha = 1, \beta > 1 \\ U(1): \text{conv. } \alpha < 1, \forall\beta \end{cases}$$

Asintoti

$\exists$ obliquo sse $\exists$ m, q

$$m = \lim_{x\to+\infty} \frac{F(x)}{x} \quad (\text{D.H.})$$

$$q = \lim_{x\to+\infty} \int (f(t) - m)\, dt$$

se $\int^{+\infty}$ e

$$q = \lim_{x\to+\infty} \{F(x) - mx\} = am + \lim \int^x (f(t) - m)$$

$= a$ o β

De l'Hopital

$$\frac{1}{|x-x_0|^\alpha} \begin{cases} U(\infty): \text{conv. } \alpha > 1 \\ U(x = x_0): \text{conv. } \alpha < 1 \end{cases}$$

Derivate

$\forall x \in D_f$

$f(x)$	$f'(x)$
c	0
x^a	$a x^{a-1}$
e^x	e^x
$\ln x$	$1/x$
$\log_a x$	$1/x \ln a$
$\log\|x\|$	$1/x$
$\log_a\|x\|$	$1/x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$tg\, x$	$1+(tg\, x)^2 = 1/(\cos x)^2$
$\arcsin x$	$1/\sqrt{1-x^2}$
$\arccos x$	$-1/\sqrt{1-x^2}$
$atg\, x$	$1/(1+x^2)$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$1-(\tanh x)^2 = 1/(\cosh x)^2$
$\|x\|$	$sig(x)$
a^x	$a^x \ln a$

$$\left(f(x)^{g(x)}\right)' = f(x)^{g(x)} \cdot g'(x) \ldots$$
$$f^g \left(g' \log f + g\, f'/f\right)$$

$$\left(f(x)\, g(x)\right)' = f'(x)\, g(x) + f(x)\, g'(x)$$

$$\left(f(x)/g(x)\right)' = \frac{f'(x)\, g(x) - f(x)\, g'(x)}{(g(x))^2}$$

$$\left(f^{-1}(y)\right)' = \frac{1}{f'(f^{-1}(y))} = \frac{1}{f'(x)}$$

con $y = f(x)$, $f'(x) \neq 0$

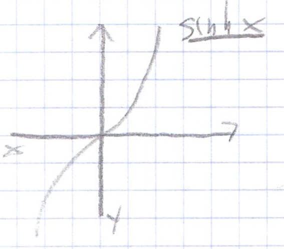
Integrali

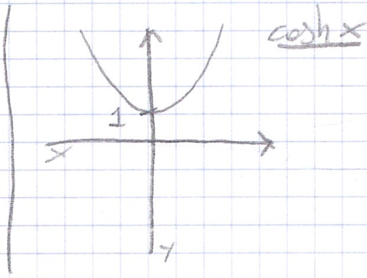
$+c \; (\in \mathbb{R})$

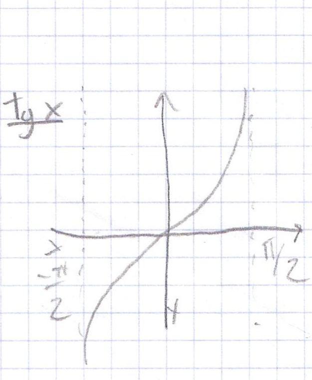
$$\int dx = x + c$$
$$\int x^a dx = \frac{x^{a+1}}{a+1} + c$$
$$\int a^x dx = \frac{a^x}{\log a} + c$$
$$\int e^x dx = e^x + c$$
$$\int \sin x\, dx = -\cos x + c$$
$$\int \cos x\, dx = \sin x + c$$
$$\int \sinh x\, dx = \cosh x + c$$
$$\int \cosh x\, dx = \sinh x + c$$
$$\int \tan x\, dx = -\log|\cos x| + c$$
$$\int \tanh x\, dx = \log(\cosh x) + c$$
$$\int \frac{1}{1+x^2} dx = atg\, x + c$$
$$\int 1/(\cos x)^2 dx = \tan x + c$$
$$\int 1/\sin^2 x\, dx = -cotg\, x + c$$
$$\int 1/\sqrt{1-x^2}\, dx = \arcsin x + c$$
$$\int 1/\sqrt{x^2-1}\, dx = \begin{cases} \log(x+\sqrt{x^2-1}) + c \\ \log(-x-\sqrt{x^2-1}) + c \end{cases}$$

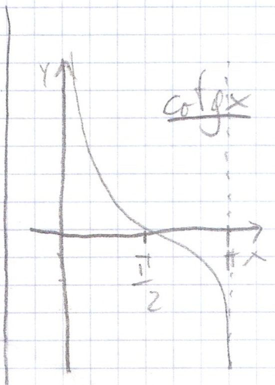
$$\int f(x)\, g'(x) = f(x)\, g(x) - \int f'(x)\, g(x)$$

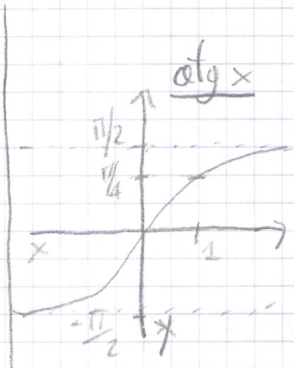
$$\int \frac{f'(x)}{k^2 + [f(x)]^2} dx = \frac{1}{k}\, atg \frac{f(x)}{k} + c$$

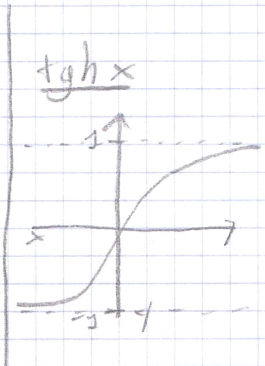












Limiti

$\lim_{x\to0} \frac{\sin x}{x} = 1$; $\sin x \sim x$; $\sin x = x + o(x)$

$\lim_{x\to0} \frac{\tan x}{x} = 1$; $\tan x \sim x$; $\tan x = x + o(x)$

$\lim_{x\to0} \frac{1-\cos x}{x^2} = \frac{1}{2}$; $1-\cos x \sim \frac{1}{2}x^2$

$\lim_{x\to0} \frac{\operatorname{arctg} x}{x} = 1$; $\operatorname{arctg} x \sim x$

$\lim_{x\to0} \frac{\arcsin x}{x} = 1$; $\arcsin x \sim x$

$\lim_{x\to0} (1+\alpha x)^{1/x} = e^{\alpha}$; $(1+\alpha x)^{1/x} \sim e^{\alpha}$

$\lim_{x\to0} \frac{\log(1+x)}{x} = 1$; $\log(1+x) \sim x$

$\lim_{x\to0} \frac{\log_a(1+x)}{x} = 1/\log a$

$\lim_{x\to0} \frac{e^x-1}{x} = 1$; $e^x - 1 \sim x$

$\lim_{x\to0} \frac{a^x-1}{x} = \log a$; $a^x - 1 \sim x\log a$

$\lim_{x\to0} \frac{(1+x)^{\alpha}-1}{x} = \alpha$; $(1+x)^{\alpha} - 1 \sim \alpha x$

$\lim_{x\to0} \frac{\sinh x}{x} = 1$; $\sinh x \sim x$

$\lim_{x\to0} \frac{\tanh x}{x} = 1$; $\tanh x \sim x$

$\lim_{x\to0} \frac{\cosh x - 1}{x^2} = \frac{1}{2}$; $\cosh x - 1 \sim \frac{1}{2}x^2$

$\lim_{x\to0} x^{\alpha}|\log(x)|^{\beta} = 0 \quad \forall \alpha,\beta>0$ | $f(x)^{\log(x)} = e^{[\log(x)\cdot\log(f(x))]}$

Formule di Mc-Laurin $(x\to0)$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + o(x^n)$$

$$a^x = \sum_0^n \frac{(x\log a)^j}{j!} + o(x^n) \quad (a>0,\ a\neq1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + (-1)^n \frac{x^{2n}}{n!} + o(x^{2n+1})$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2n}}{n!} + o(x^{2n+1})$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + (-1)^{n-1}\frac{x^n}{n} + o(x^n)$$

Serie

Geometrica

$$\sum_{n=0}^{+\infty} q^n \begin{cases} \text{Div. } +\infty & \text{se } q\geq1 \\ \text{conv. } 1/(1-q) & \text{se } |q|<1 \\ \text{irreg.} & \text{se } q\leq-1 \end{cases}$$

Mengoli $\sum_{n=1}^{+\infty} \frac{1}{n(n+1)} = 1$

- $\sum_{n=1}^{+\infty} 1/n^{\alpha} \begin{cases} \text{conv.} & \alpha>1 \\ \text{div } +\infty & \alpha\leq1 \end{cases}$
- $\sum_{n=1}^{+\infty} \frac{(-1)^n}{n^{\alpha}} \begin{cases} \text{conv. ass.} & \alpha>1 \\ \text{conv.} & 0<\alpha\leq1 \\ \text{irr.} & \alpha\leq0 \end{cases}$
- $\sum_{n=2}^{+\infty} \frac{1}{n^{\alpha}(\log n)^{\beta}} \begin{cases} \text{conv.} & \begin{cases} \alpha>1,\ \forall\beta \\ \alpha=1\ \beta>1 \end{cases} \\ \text{div.} & \begin{cases} \alpha=1,\ \beta\leq1 \\ \alpha<1,\ \forall\beta \end{cases} \end{cases}$

Confronto $0\leq a_n\leq b_n \quad \forall n\geq n_0$

$\sum b_n$ converge $\Rightarrow \sum a_n$ converge

$\sum a_n$ diverge $\Rightarrow \sum b_n$ diverge

Confronto Asintotico

$a_n, b_n > 0 \quad \forall n\geq n_0 \quad \lim_{n\to+\infty} \frac{a_n}{b_n} = L \in (0,+\infty)$

$\Rightarrow \sum a_n$ e $\sum b_n$ hanno stesso carattere

Convergenza Assoluta

$\sum|a_n|$ converge $\Rightarrow \sum a_n$ converge

Leibniz

$\sum(-1)^n a_n$ conv se $\begin{cases} a_n>0 & \forall n\geq n_0 \\ \lim_{n\to+\infty} a_n = 0 \\ a_n\geq a_{n+1} & \forall n\geq n_0 \end{cases}$

$$\operatorname{arctg} x = x - \frac{x^3}{3} + \frac{x^5}{5} + \dots + (-1)^n\frac{x^{2n+1}}{(2n+1)} + o(x^{2n+2})$$

$$(1+x)^{\alpha} = 1 + \binom{\alpha}{1}x + \binom{\alpha}{2}x^2 + \dots + \binom{\alpha}{n}x^n + o(x^n)$$

$$\binom{\alpha}{j} = \frac{\alpha(\alpha-1)(\alpha-2)\dots(\alpha-j+1)}{j!}$$

$$\frac{1}{1+x} = 1 - x + x^2 + \dots + (-1)^n x^n + o(x^n)$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + o(x^4)$$

$$\arcsin x = x + \frac{1}{2}\frac{x^3}{3} + \frac{3}{8}\frac{x^5}{5} + \frac{15}{48}\frac{x^7}{7} + o(x^8)$$

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{7}{315}x^7 + o(x^8)$$

$$\tanh x = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{7}{315}x^7 + o(x^8)$$

$Sh(x) = \frac{e^x - e^{-x}}{2}$

$Ch(x) = \frac{e^x + e^{-x}}{2}$

$Th(x) = \frac{e^{2x} - 1}{e^{2x} + 1}$

$arSh(x) = \ln(x + \sqrt{x^2+1})$

$arCh(x) = \ln(x + \sqrt{x^2-1})$

$Ch^2 - Sh^2 = 1$

$a_n = o(b_n) \underset{n \to \infty}{\iff} \frac{a_n}{b_n} = 0$

Biunivoca

- Invertibile ($f'(x) > 0 \quad \forall x \in \mathbb{R}$)
- Suriettiva ($f(\mathbb{R}) = \mathbb{R}$; $\lim_{x \to \pm\infty} f(x) = \pm\infty$)

Approssimazione di Stirling

$n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$

Taylor

usare in limiti tipo $(\to 0)$

~~con~~ $\sin(x)$
$x \to \pi$

Iniettività

f derivabile in (a, b) ; $f' > 0 \quad \forall x \in (a,b)$
$\Rightarrow f$ iniettiva in (a, b)

Codominio

$f \in C(D_f) \Rightarrow E = f(D_f)$ connesso

Sviluppi di Taylor

$$f(x)\Big|_a = f(a) + \frac{f^{(1)}(a)}{1!}(x-a)^1 + \frac{f^{(2)}(a)}{2!}(x-a)^2 + \dots + o((x-a)^n)$$

$z \to +\infty \quad z^a \ll e^z, \ \forall a > 0$